Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2014

Marking Scheme

Applied Mathematics

Ordinary Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

General Guidelines

1 Penalties of three types are applied to candidates' work as follows:

Slips	- numerical slips	S(-1)
Blunders	- mathematical errors	B(-3)
Misreading	- if not serious	M(-1)

Serious blunder or omission or misreading which oversimplifies:
- award the attempt mark only.

Attempt marks are awarded as follows: 5 (att 2), 10 (att 3).

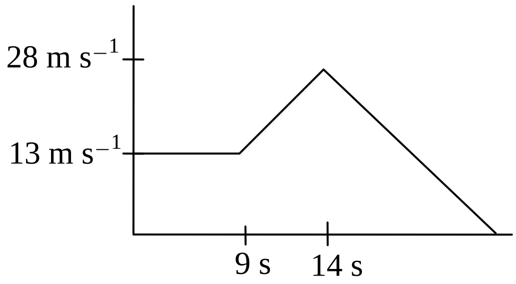
2 The marking scheme shows one correct solution to each question. In many cases there are other equally valid methods.

1. The points P and Q lie on a straight level road.
A car passes point P with a constant speed of 13 m s^{-1} and continues at this speed for 9 seconds.
The car then accelerates uniformly for 5 seconds to a speed of 28 m s^{-1}.
Finally the car decelerates uniformly from 28 m s^{-1} to rest at point Q.
The car travels 98 metres while decelerating.

(a) Draw a speed-time graph of the motion of the car from P to Q.

(b) Find **(i)** the acceleration
(ii) the deceleration
(iii) $|PQ|$, the distance from P to Q
(iv) the average speed of the car as it travels from P to Q, correct to two decimal places.

(a)



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(b) (i)

$$v = u + at$$
$$28 = 13 + a(5)$$
$$a = 3 \text{ m s}^{-2}$$

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(ii)

$$v^2 = u^2 + 2as$$
$$(0)^2 = (28)^2 + 2a(98)$$
$$a = -4 \text{ m s}^{-2}$$

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(iii)

$$s = ut + \tfrac{1}{2}at^2$$
$$s_2 = 13(5) + \tfrac{1}{2}(3)(25) = 102.5 \text{ m}$$

$$|PQ| = 13 \times 9 + 102.5 + 98 = 317.5 \text{ m}$$ 10

(iv)

$$v = u + at$$
$$0 = 28 - 4(t) \Rightarrow t = 7 \text{ s}$$ 5

$$v_1 = \frac{317.5}{14+7} = 15{\cdot}12 \text{ m s}^{-1}$$ 5

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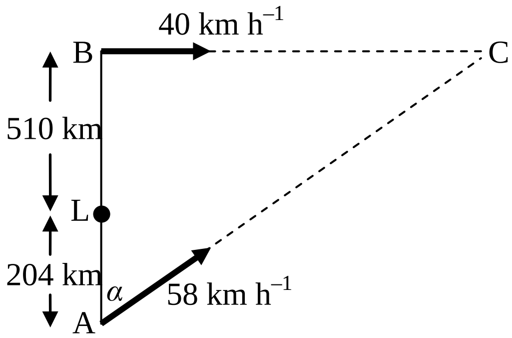
2. Ship A is positioned 204 km due south of lighthouse L.
A is moving at an angle α east of north at a constant speed of 58 km h^{-1}, where $\tan\alpha = \frac{20}{21}$.
Ship B is positioned 510 km due north of lighthouse L.
B is moving due east at a constant speed of 40 km h^{-1}.



Find **(i)** the velocity of A in terms of $\vec{i}$ and $\vec{j}$

(ii) the velocity of B in terms of $\vec{i}$ and $\vec{j}$

(iii) the velocity of A relative to B in terms of $\vec{i}$ and $\vec{j}$.

Ship A intercepts ship B after t hours.

Find **(iv)** the value of t

(v) the distance from lighthouse L to the meeting point.

Part	Solution	Marks
(i)	$\vec{V}_A = 58\sin\alpha\,\vec{i} + 58\cos\alpha\,\vec{j}$	5
	$= 40\,\vec{i} + 42\,\vec{j}$	5
(ii)	$\vec{V}_B = 40\,\vec{i} + 0\,\vec{j}$	10
(iii)	$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$	
	$= (40\,\vec{i} + 42\,\vec{j}) - (40\,\vec{i} + 0\,\vec{j})$	5
	$= 0\,\vec{i} + 42\,\vec{j}$	5
(iv)	$t = \dfrac{204 + 510}{42}$	
	$= 17$ h	10
(v)	$\lvert BC\rvert = 40 \times 17$	
	$= 680$ km	5
	$\lvert LC\rvert = \sqrt{510^2 + 680^2}$	
	$= 850$ km	5
		50

3. A particle is projected from a point on horizontal ground with an initial speed of 82 m s^{-1} at an angle β to the horizontal, where $\tan\beta = \frac{40}{9}$.

Find **(i)** the initial velocity of the particle in terms of $\vec{i}$ and $\vec{j}$
(ii) the time taken to reach the maximum height
(iii) the maximum height of the particle above ground level
(iv) the range
(v) the two times at which the height of the particle is 275 m.

(i)
$$\vec{V} = 82\cos\beta\,\vec{i} + 82\sin\beta\,\vec{j}$$
$$= 18\,\vec{i} + 80\,\vec{j}$$
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(ii)
$$v_y = u + at$$
$$0 = 80 - 10t$$
$$t = 8 \text{ s}$$
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(iii)
$$s_y = ut + \tfrac{1}{2}at^2$$
$$= 80\times 8 - 5\times 64$$
$$= 320 \text{ m}$$
10

(iv)
$$|AB| = 18\times 16$$
$$= 288 \text{ m}$$
10

(v)
$$s_y = ut + \tfrac{1}{2}at^2$$
$$275 = 80\times t - 5\times t^2$$
$$t^2 - 16t + 55 = 0$$
$$t = 5,\quad t = 11 \text{ s}$$
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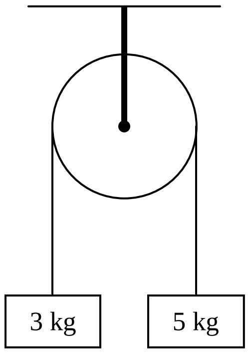
4. **(a)** Two particles of masses 3 kg and 5 kg are connected by a taut, light, inextensible string which passes over a smooth light fixed pulley.

The system is released from rest.

Find **(i)** the common acceleration of the particles

(ii) the tension in the string.



(i)

$$5g - T = 5a$$ 5

$$T - 3g = 3a$$ 5

$$2g = 8a$$

$$a = \frac{g}{4} = 2.5 \ \ \mathrm{m\,s^{-2}}$$ 5

(ii)

$$T = 3g + 3a$$

$$= 30 + 7.5$$

$$= 37.5 \ \ \mathrm{N}$$ 5

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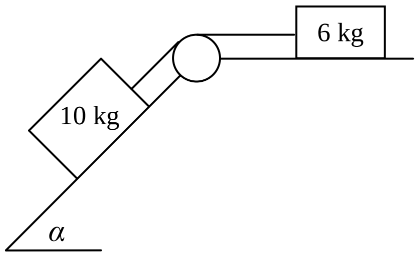
(b) Masses of 6 kg and 10 kg are connected by a taut, light, inextensible string which passes over a smooth light fixed pulley as shown in the diagram.

The 6 kg mass lies on a rough horizontal plane and the coefficient of friction between the 6 kg mass and the plane is $\frac{2}{3}$.

The 10 kg mass lies on a smooth plane which is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{4}{3}$.

The system is released from rest.

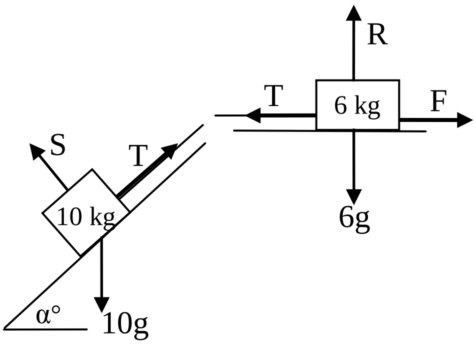


(i) Show on separate diagrams the forces acting on each particle.

(ii) Find the common acceleration of the masses.

(iii) Find the tension in the string.

(i)



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(ii)

$$10g \sin \alpha - T = 10a$$ 5

$$T - F = 6a$$ 5

$$8g - \left(\frac{2}{3}\right)6g = 16a$$

$$a = \frac{40}{16} = 2{\cdot}5 \ \text{m s}^{-2}$$ 5

(iii)

$$T = 8g - 10a$$

$$= 55 \ \text{N}$$ 5

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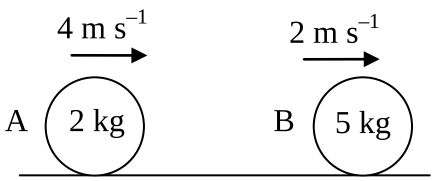
5. A smooth sphere A, of mass 2 kg, collides directly with another smooth sphere B, of mass 5 kg, on a smooth horizontal table.
A and B are moving in the same direction with speeds of 4 m s^{-1} and 2 m s^{-1} respectively.



The impulse imparted to B due to the collision is 5 N s.

Find **(i)** the speed of B after the collision

(ii) the speed of A after the collision

(iii) the coefficient of restitution for the collision

(iv) the loss in kinetic energy due to the collision.

(i)

$$I = (5)(v_2) - (5)(2) = 5$$

$$v_2 = 3 \text{ m s}^{-1}$$ 10

(ii)

$$2(4) + 5(2) = 2v_1 + 5(3)$$ 10

$$18 = 2v_1 + 15$$

$$v_1 = 1.5 \text{ m s}^{-1}$$ 5

(iii)

$$v_1 - v_2 = -e(4-2)$$

$$1.5 - 3 = -e(2)$$

$$e = \frac{3}{4}$$ 10

(iv)

$$KE_b = \tfrac{1}{2}(2)(4)^2 + \tfrac{1}{2}(5)(2)^2$$

$$= 26$$ 5

$$KE_a = \tfrac{1}{2}(2)(1.5)^2 + \tfrac{1}{2}(5)(3)^2$$

$$= 24.75$$ 5

$$KE_b - KE_a = 26 - 24.75$$

$$= 1.25 \text{ J}$$ 5

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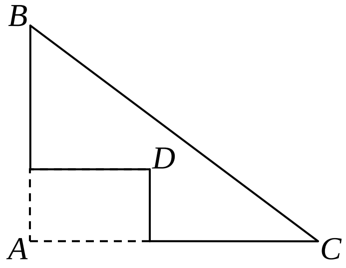
6. **(a)** Particles of weight 8 N, 2 N, 7 N and 3 N are placed at the points (6, p), (–4, q), (p, 4) and (11, 6) respectively.

The co-ordinates of the centre of gravity of the system are (4, q).

Find **(i)** the value of p

(ii) the value of q.

(b) A triangular lamina with vertices A, B and C has the rectangle with diagonal [AD] removed.
The co-ordinates of the points are $A(0, 0)$, $B(0, 18)$, $C(24, 0)$ and $D(10, 6)$.
Find the co-ordinates of the centre of gravity of the remaining lamina.



(a)

$$4 = \frac{8(6)+2(-4)+7(p)+3(11)}{20}$$ 5

$$p = 1$$ 5

$$q = \frac{8(1)+2(q)+7(4)+3(6)}{20}$$ 5

$$q = 3$$ 5

(b)

	area :	c.g.	
ABC	$\frac{1}{2}(24)(18) = 216$	$(8, 6)$	5
rectangle	$10 \times 6 = 60$	$(5, 3)$	5
lamina	$= 156$	(x, y)	5

$$(156)(x) = 216(8) - 60(5)$$

$$x = 9.15$$ 10

$$(156)(y) = 216(6) - 60(3)$$

$$y = 7.15$$ 5

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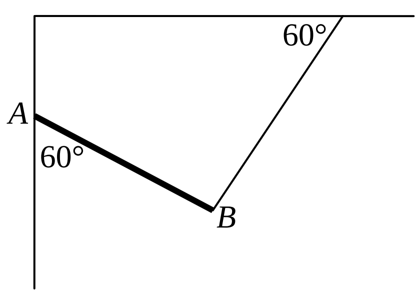
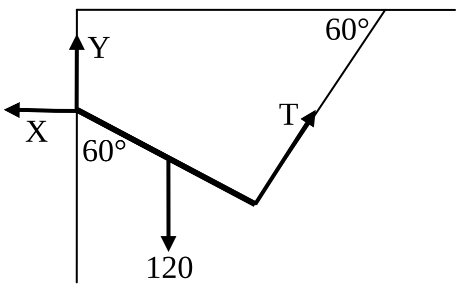
7. A uniform rod, [*AB*], of length 2 m and weight 120 N is smoothly hinged at end *A* to a vertical wall.
One end of a light inelastic string is attached to *B* and the other end of the string is attached to a horizontal ceiling.
The string makes an angle of 60° with the ceiling and the rod makes an angle of 60° with the wall, as shown in the diagram.
The rod is in equilibrium.



(i) Show on a diagram all the forces acting on the rod [*AB*].
(ii) Write down the two equations that arise from resolving the forces horizontally and vertically.
(iii) Write down the equation that arises from taking moments about the point *A*.
(iv) Find the tension in the string.
(v) Find the magnitude of the reaction at the point *A*.



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(ii) $T\cos 60 = X$ 5

$T\sin 60 + Y = 120$ 5

(iii) $T \times 2 = 120 \times 1 \sin 60$ 10

(iv) $T \times 2 = 60\sqrt{3}$

$T = 30\sqrt{3}$ 10

(v) $\left.\begin{aligned} X &= 15\sqrt{3} \\ Y &= 75 \end{aligned}\right\}$ 5

$R = \sqrt{\left(15\sqrt{3}\right)^2 + 75^2}$

$= 79.37$ N 5

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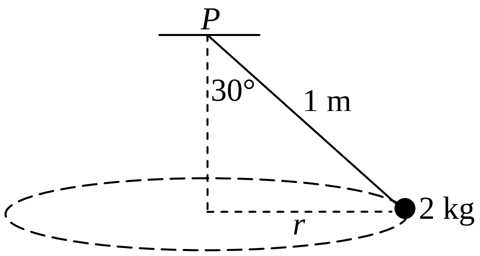
8. **(a)** A particle describes a horizontal circle of radius 2 metres with uniform angular velocity ω radians per second.
The period T (the time to travel one complete circle) is 0.4π seconds.

Find **(i)** the value of ω
(ii) the speed of the particle
(iii) the acceleration of the particle.

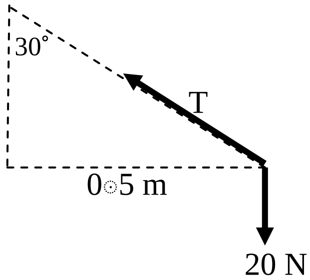
(i)	$\dfrac{2\pi}{\omega} = 0.4\pi$	
	$\omega = 5 \text{ rad s}^{-1}$	10
(ii)	$v = r\omega$	
	$= 2(5)$	
	$= 10 \text{ m s}^{-1}$	5
(iii)	$a = r\omega^2$	
	$= 2(5^2)$	
	$= 50 \text{ m s}^{-2}$	5
		$\boxed{20}$

8(b) A conical pendulum consists of a particle of mass 2 kg attached by a light inelastic string of length 1 metre to a fixed point *P*.
The string makes an angle of 30° with the vertical.

The particle describes a horizontal circle of radius *r* and the centre of the circle is vertically below *P*.



Find **(i)** the value of *r*
(ii) the tension in the string
(iii) the angular velocity of the particle.



(i)

$$\sin 30 = \frac{r}{1}$$

$$r = 0.5 \text{ m}$$ 10

(ii)

$$T\cos 30 = 20$$

$$T \times \frac{\sqrt{3}}{2} = 20 \quad \Rightarrow T = 23.09 \text{ N}$$ 10

(iii)

$$T\cos 60 = mr\omega^2$$

$$23.09 \times 0.5 = 2 \times 0.5\omega^2$$

$$\omega^2 = 11.545$$

$$\Rightarrow \omega = 3.4 \text{ rad s}^{-1}$$ 10

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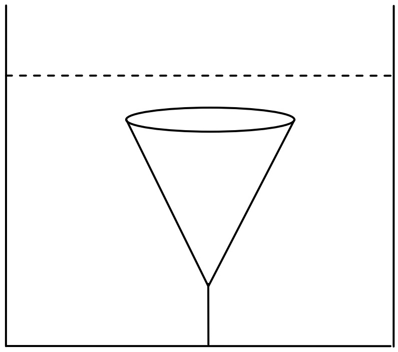
9. **(a)** State the principle of Archimedes.
A solid piece of metal has a weight of 35 N.
When it is completely immersed in water, the metal appears to weigh 27 N.
Find **(i)** the volume of the metal
(ii) the density of the metal.

(b) A right circular solid cone has a base of radius 4 cm and a height of 12 cm.
The relative density of the cone is 0.9 and it is completely immersed in a tank of liquid of relative density 1.3.
The cone is held at rest by a light, inextensible, vertical string which is attached to the base of the tank. The upper surface of the cone is horizontal.
Find the tension in the string.



(a)

Principle of Archimedes: 5

(i)

$$B = 35 - 27$$

$$\rho V g = 8$$

$$1000V(10) = 8$$

$$V = 0.0008 \text{ m}^3$$ 10

(ii)

$\rho V g = 35$	$\dfrac{W \times s_L}{s} = 8$
$\rho(0.0008)(10) = 35$	$\dfrac{35 \times 1}{s} = 8$
$\rho = 4375$	$\rho = \dfrac{35000}{8} = 4375$

10

(b)

$$B = 1300\left\{\frac{\pi}{3} \times (0.04)^2(.12)\right\}(10)$$

$$= 0.832\pi$$ 10

$$W = 900\left\{\frac{\pi}{3} \times (0.04)^2(.12)\right\}(10)$$

$$= 0.576\pi$$ 10

$$T + W = B$$

$$T = 0.832\pi - 0.576\pi$$

$$= 0.256\pi = 0.80 \text{ N}$$ 5 50

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